Vogel's Approximation method (VAM) or unit penalty method)

~~Step-1~~: The steps to be followed in this method are —

Step-1 :- Find out for each row and for each column the smallest unit cost and the next smallest unit cost. Then, determine a penalty measure for each row and each column by taking the difference (penalty) of these two and display these differences in parenthesis in the transportation table by the side of the availabilities in case of rows and below the requirements in the case of column.

Step-2 :- Identify the row or column with largest penalty and in this selected row or column allocate the maximum allowable amount to the least cell i,e, if this cell is (i, j) cell then $x_{ij} = \min(a_i, b_j)$. Adjust the supply and demand and cross out the satisfied row or column. If a row and a column is satisfied simultaneously, then cross out either row or column and assign to the remaining column or row zero demand or zero supply respectively.

~~Sol~~ Step-3 :- Repeat step 1 and step 2 for the remaining uncrossed out ~~tableau~~ tableau untill all rows and columns are crossed out.

~~Step-4~~

Ex. Use VAM to obtain an initial basic feasible solution of the T.P given below:-

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
	3	3	4	5	6	

~~Step~~

Solⁿ:-

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	3	7	4 (1)
O_2	1	4	7	2	1	8 (0)
O_3	3	9	4	8	12	9 (1)
	3	3	4	5	6	
	(1)	(5)	(3)	(1)	(6)	

Step-1

Here maximum penalty (difference) is 6 in the 5th column. The minimum cost in this column is $C_{25} = 1$

So, we take $x_{25} = \min(a_2, b_5) = \min(8, 6) = 6$

Since the demand of the column 5 is satisfied so, we cross out the fifth column. By adjusting the supplies and demands, the next uncrossed tableau is

	D_1	D_2	D_3	D_4	
O_1	2	11	10	3	4(1)
O_2	1	4	7	2	2(1)
O_3	3	9	4	8	9(1)
	3	3	4	5	
	(1)	(5)	(3)	(1)	

In the same way we get $x_{22} = 2$. proceedin in the same manner, other basic variables are given from the successive tableau.

The next tableau is

	D_1	D_2	D_3	D_4	
O_1	2	11	10	3	4(1)
O_3	3	9	4	8	9(1).
	3	1	4	5	
	(1)	(2)	(6)	(5)	

and $x_{33} = 4$. The next tableau is

	D_1	D_2	D_4	
O_1	2	11	3	4(1)
O_3	3	9	8	5(5)
	3	1	5	
	(1)	(2)	(5)	

and $x_{31} = 3$. The next tableau is

	D_2	D_4	
O_1	11	3	4 (8)
O_3	9	8	1 (1)
	1	5	
	(2)	(5)	

and $x_{14} = 4$

The next tableau is

	D_2	D_4	
O_3	9	8	2
	1	1	

and $x_{32} = 1$ and $x_{34} = 1$

The final tableau is as follows

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	[4] 3	7	4
O_2	1	[2] 4	7	2	[6] 1	8
O_3	[3] 3	[1] 9	[4] 4	[1] 8	12	9
	3	3	4	5	6	

The number of occupied cells = 7 $(m+n-1)$ and this set of cells does not contain any loop. So, a basic feasible solution is

$x_{14} = 4$; $x_{22} = 2$; $x_{25} = 6$; $x_{31} = 3$; $x_{32} = 1$
$x_{33} = 4$; $x_{34} = 1$

cost $= (4\times3) + (2\times4) + (6\times1) + (3\times3) + (1\times9) + (4\times4)$
$+ (1\times8)$ unit
$= 68$ unit.

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Note: The whole computations can be shown in a single tableau in the following manner—

	D_1	D_2	D_3	D_4	D_5	a_i						
O_1	2	11	10	$\boxed{4}$ 3	7	4(1)	4(1)	4(1)	4(1)	4(8)		
O_2	1	$\boxed{2}$ 4	7	2	$\boxed{6}$ 1	8(0)	2(1)					
O_3	$\boxed{3}$ 3	$\boxed{1}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	12	9(1)	9(1)	9(1)	5(5)	2(1)	2	
	3 (1)	3 (5)	4 (3)	5 (1)	6 (6)							
	3 (1)	3 (5)	4 (3)	5 (1)								
	3 (1)	1 (2)	4 (6)	5 (5)								
	3 (1)	1 (2)		5 (5)								
		1 (2)		5 (5)								
		1		1								

~~Determine~~ solve by V.A.M

	D_1	D_2	D_3	D_4	
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
	6	10	12	15	

Solution :- Since $\sum a_i = \sum b_j = 43$, so it is a balan[ced] T.P. An initial basic feasible solution by VAM shown in the following table :-

	D_1	D_2	D_3	D_4					
1	21	16	25	$\boxed{11}$ 13	11 (3)				
O_2	$\boxed{6}$ 17	$\boxed{3}$ 18	14	$\boxed{4}$ 23	13 (3)	13 (3)	9 (3)	3 (4)	3
O_3	32	$\boxed{7}$ 27	$\boxed{12}$ 18	41	19 (9)	19 (9)	19 (9)	19 (9)	7
	6 (4)	10 (2)	12 (4)	15 (10)					
	6 (15)	10 (9)	12 (4)	4 (18)					
	6 (15)	10 (9)	12 (4)						
		10 (9)	12 (4)						
		10							

The sol^n will be $x_{14} = 11$; $x_{21} = 6$; $x_{22} = 3$; $x_{24} = 4$; $x_{32} = 7$; $x_{33} = 12$

The corresponding cost $= (11 \times 13) + (6 \times 17) + (3 \times 18) + (4 \times 23) + (7 \times 27) + (12 \times 18)$

$= 796$ unit.

Solve by VAM

H-W

	A	B	C	a_i
F_1	10	9	8	8
F_2	10	7	10	7
F_3	11	9	7	9
F_4	12	14	10	4
b_j	10	10	8.	

Sol Though solution is not unique here but one I.B.F.S $x_{11}=6$; $x_{12}=2$; $x_{22}=7$; $x_{32}=1$; $x_{33}=8$; $x_{41}=4$

cost = 240 unit.